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# Pearson Edexcel Level 3 GCE

**Tuesday 4 June 2024**

Afternoon (Time: 2 hours) | Paper reference **9MA0/01**

# Mathematics

**Advanced**

**PAPER 1: Pure Mathematics 1**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.**

Notes:

Completing a colour coded section typifies achieving this grade. Dropped marks can be made up elsewhere.

To achieve a grade D would require achieving the same marks as the Grade E and D sections, Grade C would require achieving the same marks as the grade E, D and C sections.

Partial completion of a section would imply working towards this grade.

Blank pages have been removed form this scan where additional working out space would be available.

Colour codes

Grade E Grade D Grade C Grade B Grade A

**1.** $$g(x) = 3x^3 - 20x^2 + (k + 17)x + k$$

where $k$ is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of $k$.

**(3)**

$g(3) = 0$

$3(3)^3 - 20(3)^2 + (k+17)3 + k = 0$

$81 - 180 + 3k + 51 + k = 0$

$4k = 48$

$\underline{k = 12}$

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2. (a) Find, in ascending powers of $x$, the first four terms of the binomial expansion of

$$(1-9x)^{\frac{1}{2}}$$

giving each term in simplest form.

**(3)**

(b) Give a reason why $x = -\frac{2}{9}$ should **not** be used in the expansion to find an approximation to $\sqrt{3}$

**(1)**

a/ $1 + \frac{1}{2}(-9x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-9x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{6}(-9x)^3$

$1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$

b/ The expansion is valid when $|9x| < 1$

$x < |\frac{1}{9}|$

$x$ has to be between $-\frac{1}{9}$ and $\frac{1}{9}$

$\therefore$ the expansion is not valid for $x = \frac{2}{9}$

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3. $$f(x) = x + \tan\left(\frac{1}{2}x\right) \qquad \pi < x < \frac{3\pi}{2}$$

Given that the equation $f(x) = 0$ has a single root $\alpha$

(a) show that $\alpha$ lies in the interval [3.6, 3.7]

**(2)**

(b) Find $f'(x)$

**(2)**

(c) Using 3.7 as a first approximation for $\alpha$, apply the Newton–Raphson method once to obtain a second approximation for $\alpha$. Give your answer to 3 decimal places.

**(2)**

a/ $x + \tan(\frac{1}{2}x) = 0$

$3.6 + \tan(\frac{1}{2} \cdot 3.6) = -0.686$

$3.7 + \tan(\frac{1}{2} \cdot 3.7) = 0.212$

change of sign and the function is continuous in the interval ∴ there is a root in the interval [3.6, 3.7]

b/ $f(x) = x + \tan(\frac{1}{2}x)$

$f'(x) = 1 + \frac{1}{2}\sec^2(\frac{1}{2}x)$

c/ $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = 3.7$

$$x_2 = 3.7 - \frac{3.7 + \tan(\frac{1}{2} \cdot 3.7)}{1 + \frac{1}{2}\sec^2(\frac{1}{2} \cdot 3.7)}$$

$$= 3.7 - \frac{0.212}{7.58}$$

$$= 3.672$$



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**Question 2 continued**

**(Total for Question 2 is 4 marks)**



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Turn over

**4.** Given that $y = x^2$, use differentiation from first principles to show that $\frac{dy}{dx} = 2x$

**(3)**

$f(x) = x^2$

$f(x+h) = (x+h)^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h)$$

$$= 2x$$

Grade E

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5. The function f is defined by

$$f(x) = \frac{2x-3}{x^2+4} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2+bx+c}{\left(x^2+4\right)^2}$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

(b) Hence, using algebra, find the values of $x$ for which f is decreasing.
You must show each step in your working.

**(3)**

a/ $u = 2x - 3 \qquad v = x^2 + 4$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = 2x$

$$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$$

$$= \frac{2x^2 + 8 - 4x^2 + 6x}{(x^2+4)^2}$$

$$= \frac{-2x^2 + 6x + 8}{(x^2+4)^2}$$

Grade E

b/ $\frac{-2x^2+6x+8}{(x^2+4)^2} < 0$

Working on grade D

$-2x^2 + 6x + 8 < 0$

$\underline{\underline{x < -1}}$ or $\underline{\underline{x > 4}}$



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**6.**

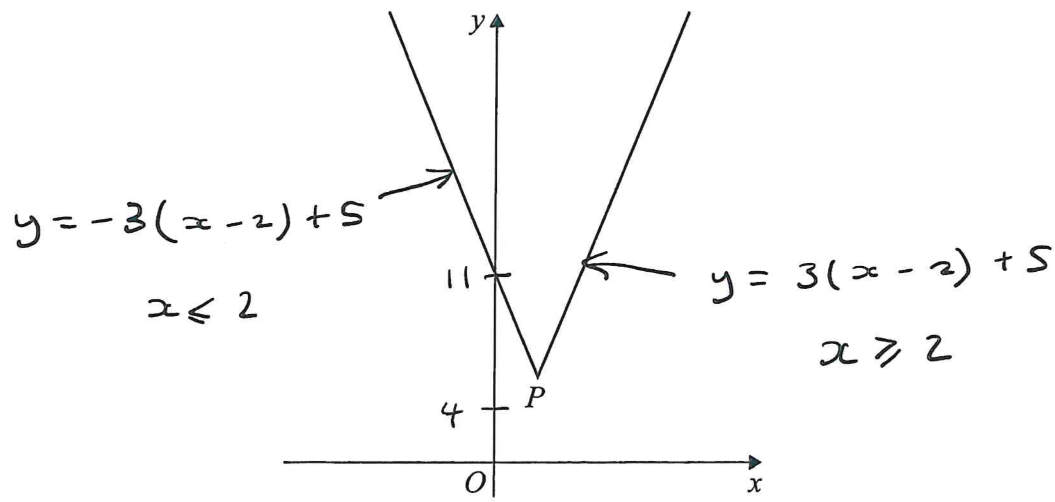


**Figure 1**

Figure 1 shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point $P$, shown in Figure 1.

(a) Find the coordinates of $P$.

**(2)**

(b) Solve the equation

$$16 - 4x = 3|x - 2| + 5$$

**(2)**

A line $l$ has equation $y = kx + 4$ where $k$ is a constant.

Given that $l$ intersects $y = 3|x - 2| + 5$ at 2 distinct points,

(c) find the range of values of $k$.

**(2)**

a/ $(2, 5)$

b/ $16 - 4x = 3(x - 2) + 5$ $\qquad$ $16 - 4x = -3(x - 2) + 5$

$16 - 4x = 3x - 6 + 5$ $\qquad$ $16 - 4x = -3x + 6 + 5$

$17 = 7x$ $\qquad$ $5 = x$

$x = \frac{17}{7}$ $\qquad$ ✗

$x \leq 2$ so $x = 5$ is not an intersection



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**Question 6 continued**

c/ one intersection if the line passes through (2, 5)

$y = kx + 4$

$5 = 2(k) + 4$

$1 = 2k$

$k = \frac{1}{2}$

only 1 intersection if $k = 3$

$\therefore \frac{1}{2} < k < 3$

**(Total for Question 6 is 6 marks)**



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**7.**

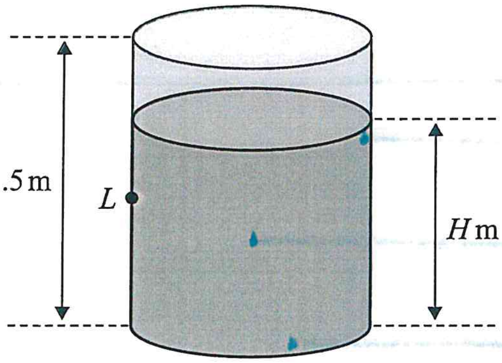


**Figure 2**

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point $L$, in the side of the tank.

While the tank is leaking, the depth, $H$ metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where $t$ hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where $A$ and $B$ are constants to be found,

**(3)**

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

**(3)**

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

**(2)**

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

$$\int 1 \, dH = \int -0.12e^{-0.2t} \, dt$$



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**Question 7 continued**

$$H = 0.6e^{-0.2t} + C$$

when $t=0$ $H=1.5$

$$1.5 = 0.6 + C$$
$$C = 0.9$$

$$H = 0.6e^{-0.2t} + 0.9$$

b/ $1.2 = 0.6e^{-0.2t} + 0.9$

$$0.3 = 0.6e^{-0.2t}$$
$$\frac{1}{2} = e^{-0.2t}$$
$$\ln\frac{1}{2} = -0.2t$$
$$t = 3.47 \text{ hours}$$
$$= \underline{\underline{3 \text{ hours } 28 \text{ minutes}}}$$

c/ As $t$ increases $H$ gets closer to $0.9$

$\underline{\underline{0.9\text{m}}}$

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Turn over

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**8.** The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x-9} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

(a) Find fg(2)

**(2)**

(b) Find $g^{-1}$

**(3)**

(c) (i) Find gf($x$), giving your answer as a simplified fraction.

(ii) Deduce the range of gf($x$).

**(3)**

The function h is defined by

$$h(x) = 2x^2 - 6x + k \qquad x \in \mathbb{R}$$

where $k$ is a constant.

(d) Find the range of values of $k$ for which the equation

$$f(x) = h(x)$$

has no real solutions.

**(3)**

a/ $g(2) = \frac{5}{2(2)-9}$

$= -1$

$f(-1) = 4 - 3(-1)^2$

$= 1$

b/ $y = \frac{5}{2x-9}$

$y(2x-9) = 5$

$2xy - 9y = 5$

$2xy = 5 + 9y$

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**Question 8 continued**

$$x = \frac{5+9y}{2y}$$

$$g^{-1}(x) = \frac{5+9x}{2x} \qquad x \in R \quad x \neq 0$$

Grade D

c i) $gf(x) = \dfrac{5}{2(4-3x^2)-9}$

$= \dfrac{5}{8-6x^2-9}$

$= \dfrac{5}{-6x^2-1}$

$= -\dfrac{5}{6x^2+1}$

Working on Grade C

ii) The smallest the denominator can be is 1

when $x=0$ $\quad gf(x) = -5$

As $x$ increases $gf(x)$ gets closer to zero

$$\underline{\underline{-5 \leq gf(x) < 0}}$$

d) $4-3x^2 = 2x^2 - 6x + k$

$0 = 5x^2 - 6x + (k-4)$

No solutions where $b^2 - 4ac < 0$



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**Question 8 continued**

$(-6)^2 - 4(5)(k-4) < 0$

$36 - 20(k-4) < 0$

$36 - 20k + 80 < 0$

$116 < 20k$

$5.8 < k$

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**9.** The first 3 terms of a geometric sequence are

$$3^{4k-5} \qquad 9^{7-2k} \qquad 3^{2(k-1)}$$

$$3^{2(7-2k)}$$

where $k$ is a constant.

(a) Using algebra and making your reasoning clear, prove that $k = \dfrac{5}{2}$ **(3)**

(b) Hence find the sum to infinity of the geometric sequence. **(3)**

$$\frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2(k-1)}}{3^{2(7-2k)}}$$

$$3^{2(7-2k)-(4k-5)} = 3^{2(k-1)-2(7-2k)}$$

$$14 - 4k - 4k + 5 = 2k - 2 - 14 + 4k$$

$$19 - 8k = 6k - 16$$

$$35 = 14k$$

$$k = \frac{35}{14} = \frac{5}{2}$$

b/ $a = 3^{4(\frac{5}{2})-5}$ $\qquad u_2 = 9^{7-2(\frac{5}{2})}$

$= 81$

$= 243$

$$r = \frac{81}{243} = \frac{1}{3}$$

$$S_\infty = \frac{243}{1-\frac{1}{3}}$$

Grade C

$$= \frac{729}{2}$$



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**10.**

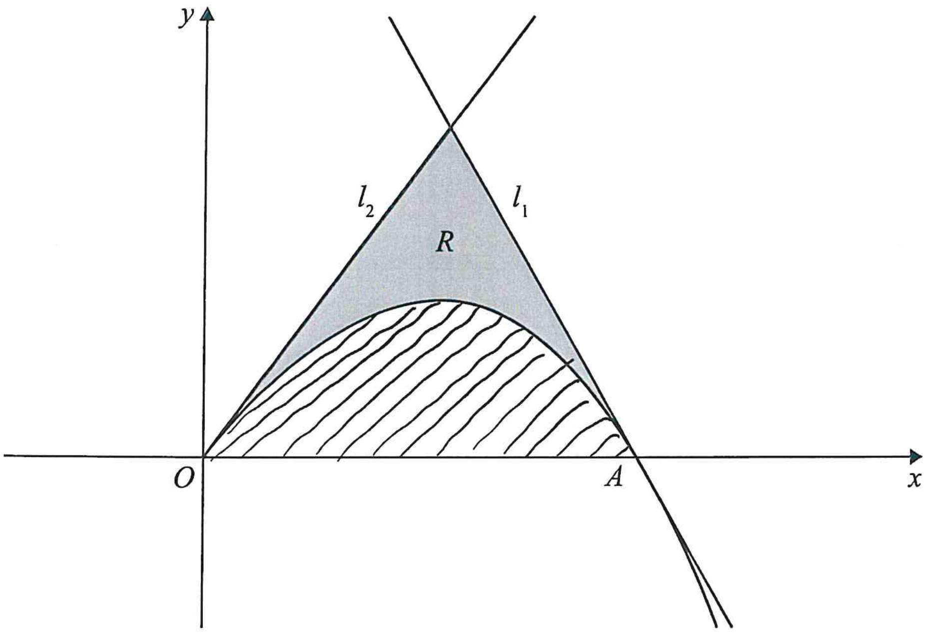


**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the $x$-axis at the point $A$.

(a) Verify that the $x$ coordinate of $A$ is 4

**(1)**

The line $l_1$ is the tangent to the curve at $A$.

(b) Use calculus to show that an equation of line $l_1$ is

$$12x + y = 48$$

**(3)**

The line $l_2$ has equation $y = 8x$

The region $R$, shown shaded in Figure 3, is bounded by the curve, the line $l_1$ and the line $l_2$

(c) Use algebraic integration to find the exact area of $R$.

**(5)**

a) $0 = 8(4) - 4^{\frac{5}{2}}$

$0 = 0$ ✓



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**Question 10 continued**

Working on Grade B

b/ $y = 8x - x^{\frac{5}{2}}$

$\frac{dy}{dx} = 8 - \frac{5}{2}x^{\frac{3}{2}}$

when $x = 4$ $\frac{dy}{dx} = 8 - \frac{5}{2}(4)^{\frac{3}{2}}$

$= -12$

(4, 0)

$x$ $y$ $y - 0 = -12(x - 4)$

$y = -12x + 48$

$\underline{12x + y = 48}$

c/ $y = 8x$ $y = -12x + 48$

$l_1$ and $l_2$ intersect where $8x = -12x + 48$

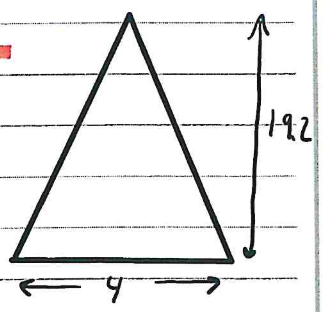
$20x = 48$

$x = 2.4$

when $x = 2.4$ $y = 8(2.4) = 19.2$

Area of triangle $= \frac{1}{2} \cdot 4 \cdot 19.2$

$= 38.4$ units$^2$




$\int_0^4 8x - x^{\frac{5}{2}} \, dx$

$\left[4x^2 - \frac{2}{7}x^{\frac{7}{2}}\right]_0^4$



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**Question 10 continued**

$$\left(4(4)^2 - \frac{2}{7}(4)^{\frac{7}{2}}\right) - (0) = \frac{192}{7}$$

$$38.4 - \frac{192}{7} = \frac{384}{35} \text{ units}^2$$

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**11.**

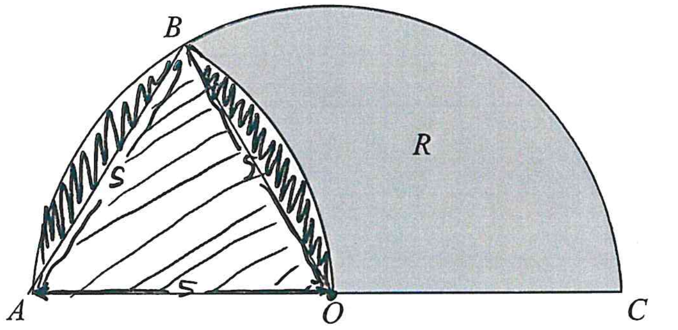


**Figure 4**

Figure 4 shows the design of a badge.

The shape $ABCOA$ is a semicircle with centre $O$ and diameter 10 cm.

$OB$ is the arc of a circle with centre $A$ and radius 5 cm.

The region $R$, shown shaded in Figure 4, is bounded by the arc $OB$, the arc $BC$ and the line $OC$.

Find the exact area of $R$.

Give your answer in the form $(a\sqrt{3} + b\pi)\,\text{cm}^2$, where $a$ and $b$ are rational numbers.

**(4)**

Area of semi circle $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(5)^2(\pi)$

$= \frac{25}{2}\pi$

Area of triangle $= \frac{1}{2}(5)(5)\sin\left(\frac{\pi}{3}\right)$

$= \frac{25\sqrt{3}}{4}$

Area of segment $= \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right) - \frac{25\sqrt{3}}{4}$

$= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

$R = \frac{25}{2}\pi - \frac{25\sqrt{3}}{4} - 2\left(\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}\right)$

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**Question 11 continued**

$$= \frac{25}{2}\pi - \frac{25\sqrt{3}}{4} - \frac{25}{3}\pi + \frac{25}{2}\sqrt{3}$$

$$= \frac{25}{6}\pi + \frac{25}{4}\sqrt{3} \text{ cm}^2$$

Grade B

Working on Grade A

**(Total for Question 11 is 4 marks)**

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**12.** (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of $K$ and give the value of $\alpha$, in degrees, to 2 decimal places.

**(3)**

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, $R$, is modelled by the equation

$$R = A + 140\cos(30t)° - 480\sin(30t)°$$

where $t$ months is the time after the start of the year and $A$ is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

**(2)**

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

**(2)**

The number of foxes, $F$, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)°$$

The number of foxes is at its minimum value after $T$ months.

(d) Find, according to the models, the number of **rabbits** in the wood at time $T$ months.

**(4)**

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

Working on Grade A

$K\cos(\theta+\alpha) = K\cos\theta\cos\alpha - K\sin\theta\sin\alpha$

$= 140\cos\theta - 480\sin\theta$

$K = \sqrt{140^2 + 480^2}$ $\quad K\sin\alpha = 480$

$= \underline{500}$ $\quad K\cos\alpha = 140$

$\tan\alpha = \frac{480}{140}$

$\alpha = \tan^{-1}\left(\frac{480}{140}\right)$



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Question 12 continued

$\alpha = 73.74°$

$500 \cos(\theta + 73.74)$

b/ i/ $R = A + 500 \cos(30t + 73.74)$

as max $= 1500$ $A = 1000$

$R = 1000 + 500 \cos(30t + 73.74)$

ii/ min $1000 - 500 = 500$

c/ min when $\cos(30t + 73.74) = -1$

$30t + 73.74 = 180$

$t = 3.54$

3.5 months after the start of the year is mid April $\therefore$ the model is suitable.

$F = 100 + 70 \sin(30t + 70)$

min when $\sin(30t + 70) = -1$

$30t + 70 = -90, 270$

$30t = -160, 200$

$t = \frac{20}{3}$ months

$R = 1000 + 500 \cos(30(\frac{20}{3}) + 73.74)$

$= 1033$

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**13.** (a) Given that $a$ is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$$

**(4)**

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = k\pi a^2$$

where $k$ is a constant to be found.

**(4)**

$x = a\sin^2\theta$ $\qquad \frac{dx}{d\theta} = 2a\sin\theta\cos\theta$

when $x = a$ $\quad a = a\sin^2\theta$

$1 = \sin^2\theta$

$1 = \sin\theta$

$\theta = \frac{\pi}{2}$

$\boxed{\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ a\sin^2\theta + a\cos^2\theta = a\end{array}}$

when $x = 0$ $\quad 0 = a\sin^2\theta$

$0 = \sin\theta$

$\theta = 0$

$$\int_0^{\frac{\pi}{2}} (a\sin^2\theta)^{\frac{1}{2}}\sqrt{a - a\sin^2\theta}\,\frac{dx}{d\theta}\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \sqrt{a}\sin\theta\cdot\sqrt{a\cos^2\theta}\cdot 2a\sin\theta\cos\theta\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \sqrt{a}\sin\theta\cdot\sqrt{a}\cos\theta\cdot 2a\sin\theta\cos\theta\,d\theta$$

$$\int_0^{\frac{\pi}{2}} 2a^2\sin^2\theta\cos^2\theta\,d\theta$$

$$2a^2\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\,d\theta$$



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Question 13 continued

$$\cancel{\sin 2A = 2\sin A\cos A}$$

$$\cancel{\frac{1}{2}\sin 2A = \sin A\cos A}$$

$$\cancel{\frac{1}{4}\sin^2 2A = \sin^2 A\cos^2 A}$$

$$\cancel{2a^2\int_0^{\frac{\pi}{2}}\frac{1}{4}\sin^2 2\theta \, d\theta}$$

$$\cancel{\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta \, d\theta}$$

Grade A

b/

$$\cos 2A = 1 - 2\sin^2 A$$
$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$2\sin^2 2\theta = 1 - \cos 4\theta$$
$$\sin^2 2\theta = \frac{1}{2} - \frac{1}{2}\cos 4\theta$$

$$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\frac{1}{2} - \frac{1}{2}\cos 4\theta \quad d\theta$$

$$\frac{1}{2}a^2 \cdot \left[\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta\right]_0^{\frac{\pi}{2}}$$

$$\frac{1}{2}a^2\left[\left(\frac{1}{4}\pi - 0\right) - (0 - 0)\right]$$

$$\frac{1}{8}\pi a^2$$

**14.** A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time $t$ seconds, the radius of the balloon is $r$ cm.

(a) Write down a differential equation to model this situation. **(1)**

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of $0.9\,\text{cm s}^{-1}$

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

**(5)**

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre. **(2)**

(d) Suggest a limitation of the model. **(1)**

a/ $\frac{dr}{dt} = \frac{k}{\sqrt{r}}$ $\quad\quad$ $\frac{dr}{dt} = \frac{3.6}{\sqrt{r}}$

b/ $\int \sqrt{r}\, dr = \int 3.6\, dt$ $\quad\quad$ when $t = 10$ $r = 16$ and $\frac{dr}{dt} = 0.9$

$\frac{2}{3} r^{3/2} = 3.6t + c$

$0.9 = \frac{k}{\sqrt{16}}$

$\frac{2}{3}(16)^{\frac{3}{2}} = 3.6(10) + c$ $\quad\quad$ $k = 3.6$

$\frac{128}{3} = 36 + c$

$c = \frac{20}{3}$



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**Question 14 continued**

$$\frac{2}{3} r^{\frac{3}{2}} = 3.6t + \frac{20}{3}$$

$$r^{\frac{3}{2}} = 5.4t + 10$$

c/ when $t = 20$

$$r^{\frac{3}{2}} = 118$$

$$r = (118)^{\frac{2}{3}}$$

$$= 24.1 \text{ cm}$$

d/ The balloon will pop if it keeps expanding.



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**15.** (i) Show that $k^2 - 4k + 5$ is positive for all real values of $k$.

**(2)**

(ii) A student was asked to prove by contradiction that

"There are no positive integers $x$ and $y$ such that $(3x + 2y)(2x - 5y) = 28$"

The start of the student's proof is shown below.

> Assume that positive integers $x$ and $y$ exist such that
> $$(3x + 2y)(2x - 5y) = 28$$
> If $3x + 2y = 14$ and $2x - 5y = 2$
> $$\left.\begin{array}{l} 3x + 2y = 14 \\ 2x - 5y = 2 \end{array}\right\} \Rightarrow x = \frac{74}{19},\ y = \frac{22}{19} \text{ Not integers}$$

Show the calculations and statements needed to complete the proof.

**(4)**

a) $k^2 - 4k + 5$

$(k-2)^2 - 4 + 5$

$(k-2)^2 + 1$

The minimum value is 1 $\therefore$ positive for all values of $k$.

b) • As $x$ and $y$ are integers $(3x+2y)$ and $(2x - 5y)$ must be integers

• As $x$ and $y$ are positive integers: $3x+2y \geq 5$

$28 = 1 \times 28$
$= 2 \times 14$ (Already checked)
$= 4 \times 7$



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**Question 15 continued**

$3x + 2y = 28$
$2x - 5y = 1$

$x = \frac{142}{19}$ $y = \frac{53}{19}$ not integers

$3x + 2y = 7$
$2x - 5y = 4$

$x = \frac{43}{19}$ $y = \frac{2}{19}$ not integers

Therefore there are no positive integers $x$ and $y$ such that $(3x+2y)(2x-5y) = 28$

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